A Composite Random Walk for Facing Environmental Uncertainty and Reduced Perceptual Capabilities

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Abstract. Theoretical and empirical studies in Biology have showed that strategies based on different random walks, such as: Brownian random walk and Lévy random walk are the best option when there is some degree of environmental uncertainty and there is a lack of perceptual capabilities.

When a random walker has no information about where targets are located, different systematic or random searches may provide different chances to find them. However, when time consumption, energy cost and malfunction risks are determinants, an adaptive search strategy becomes necessary in order to improve the performance of the strategy. Thus, we can use a practical methodology to combine a systematic search with a random search through a biological fluctuation.

We demonstrate that, in certain environments it is possible to combine a systematic search with a random search to optimally cover a given area. Besides, this work improves the search performance in comparison with pure random walks such as Brownian walk and Lévy walk. We show these theoretical results using computer simulations.

1 Introduction

Several studies suggest that Brownian walks and Lévy walks are commonly used models to fit animal movement [1,2,3,4,5], these strategies represent the standard methods for exploring a given area where there is not available information about where targets are located; a target may be an abstraction from any particular object that must be collected by a random walker (in this study we will refer a random walker as a robotic agent).

A fast searching with reduced perceptual capabilities and some degree of uncertainty is commonly seen on exploration tasks that are carried on by mobile robots or synthetic agents [4,6]. In addition, the major aim is to maximize the chance of finding as many targets as possible.

Random walks have become a necessary tool for exploration tasks in robotics field, where researches try to mimic strategies based on animal behavior to face search uncertainties and environmental changes [4].

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Plank and Codling [7] argue that "Lévy walks can be optimal strategies for searching randomly located targets in complex environments [2,3]. However, the assertion that the Lévy searching model is superior to other movement processes (such as composite random walks) is being questioned".

In this paper, we show how a simulated composite random walk can cope with environmental uncertainty with reduced perceptual capabilities. Thus, this approach also presents an acceptable performance compared with others random searches (i.e. Brownian walk, Lévy walk and intermittent strategies).

The remainder of this paper is organized as follows. Section 2 describes two main types of search strategies: random searches and systematic searches. Section 3 explains our composite random walk using a ballistic motion strategy and sequences of knots. Subsequently, in section 4 we present preliminary experiments about our assumption. Results from these experiments are compared in section 5. Finally, section 6 concludes the paper.

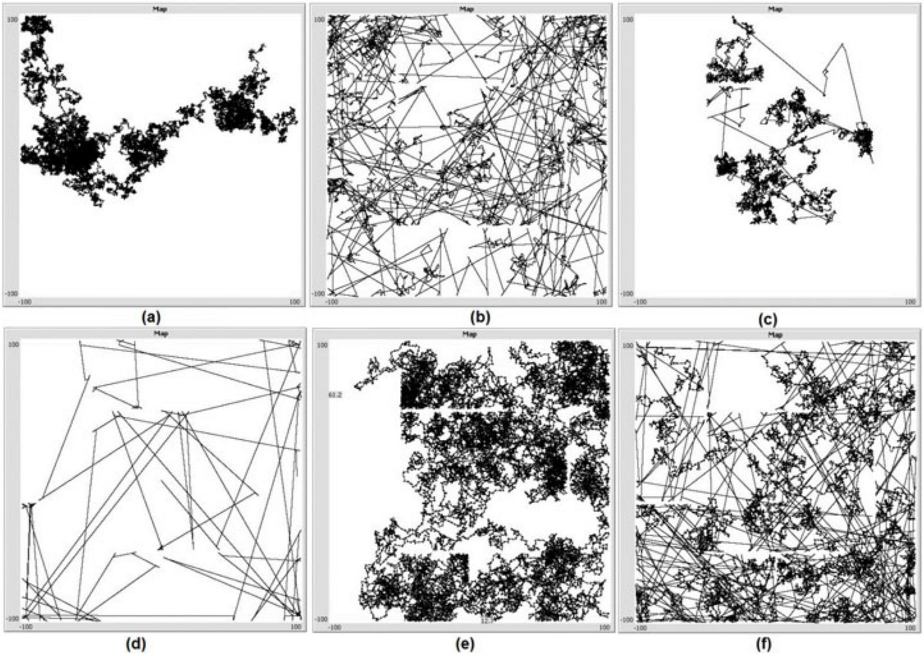
2 Methods

According to the inherent properties of the searching mechanism, it is possible to find two main types of searching strategies: *random searches* and *systematic searches.* In random searches, these rules rely on stochastic processes (i.e. the sampling of probability distributions), while, in systematic searches, the rules to optimally cover a given area are based on deterministic algorithms (i.e. fixed and organized plans) [3].

2.1 Random Searches

A random walk consists of a series of steps (possibly of different size) in randomly chosen directions [8]. Bearing this in mind, Viswanathan *et al.* [2] define a Lévy walk as a distribution function $P(l_j) \sim l_j^{-\mu}$ with $1 < \mu \leq 3$ where l_j is the flight length. The μ exponent is the Lévy index and controls the range of correlations in the movement. Thus, Lévy models comprise a rich variety of paths ranging from Brownian motion ($\mu > 3$) to straight-line paths ($\mu \to 1$) [3].

Brownian Walk. This random search is derived from the probability distribution that is specified in a Lévy walk, when the Lévy index ($\mu > 3$). The Gaussian is the stable distribution for the special case $\mu \geq 3$ owing to the central limit theorem. Where its behavior does not present a heavy tailed distribution. some remarkable features presented by a Brownian walk are as follows: explorations over short distances can be made in much shorter times than explorations over long distances, the random walker tends to explore a given region of space rather thoroughly, it tends to return to the same point many times before finally wandering away. It chooses new regions to explore blindly. The random walker has no any tendency to move toward regions that it has not occupied before; it has absolutely no inkling of the past and lastly,its track does not fill up the space uniformly [5,4]. A Brownian walk plotted in our simulator tool is depicted in Figure 1a.



(a) (b) (c) (d) (e) (f)

Fig. 1. Plots showing simulated random walks: (a) Brownian walk ($\mu = 3$); (b) Lévy walk ($\mu = 2$); (c) Adaptive switching behavior between Lévy walk and Brownian walk; (d) Intermittent strategy using regime 1; (e) Sequence of knots as a random walk; (f) Composite random walk

Lévy Walk. A Lévy movement pattern is generally assumed to be a "scale-invariant," i.e., it has the fractal property that the sampling scale used by the observer should not affect the observed properties [7]. In particular, a Lévy walk is known to be a super diffusive at all scales [2]. The trajectory of a Lévy walk comprises short walks with a turning angle step, $\phi_n = \theta_n - \theta_{n-1}$, is drawn from a uniform distribution on $[-\pi, \pi]$ [7].

A Lévy distribution is advantageous when target sites are sparsely and randomly distributed, irrespective of the value of μ chosen, because the probability of returning to a previously visited site is smaller than for a Gaussian distribution [2]. Theoretical arguments and numerical simulations suggest that ($\mu \approx 2$) is the optimal value for a search in any dimension and the solution to certain optimal foraging problems [1,2,3,4]. Figure 1b shows a plot of a simulated Lévy walk.

Adaptive Switching Behavior between Lévy Walk and Brownian Walk. Nurzaman *et al.* [4] compare the efficiency of a Lévy walk ($\mu = 2$), a Brownian walk ($\mu = 3$) with an adaptive switching between Lévy and Brownian walk based on biological fluctuation.

The biological fluctuation allows the robot can adaptively adjust its random search property based on encounters with targets in the environment. Nurzaman *et al.* [4] designed a model for representing the switching probability through a simple unimodal potential function $U(z(t)) = (z(t) - h)^2$ where $z(t)$ is a variable characterizing biological fluctuation given by

$$\dot{z}(t) = -\frac{dU(z(t))}{dz} A(t) + \epsilon(t) \tag{1}$$

$$= -2(z(t) - h)A(t) + \epsilon(t) \ . \tag{2}$$

Where h is the position of the attractor, $A(t)$ is the activity parameter and $\epsilon(t)$ is the noise term (white noise), representing the stochastic driving force. Thus, the switching probability at time t from Lévy random walk to Brownian random walk is given by

$$P(t) = \exp(-z(t)) \ . \tag{3}$$

The fitness of the environment is defined by the activity parameter $A(t)$ given by

$$A(t) = \begin{cases} A_{min}, & \text{if } \alpha(t) \le A_{min} \\ \alpha(t), & \text{if } \alpha(t) > A_{min} \ . \end{cases} \tag{4}$$

$$\alpha(t) = C\alpha(t-1) + K_F F(t) \ . \tag{5}$$

$$F(t) = \begin{cases} 1, & \text{if one or more targets are found at time t} \\ 0, & \text{if no targets are found at time t } . \end{cases} \tag{6}$$

with $0 < C < 1$ and K_F is a constant with a large value with respect to A_{min}. Figure 1c shows a plot of a simulated adaptive switching behavior between Lévy walk and Brownian walk.

Intermittent Strategy or "Saltatory". Bénichou *et al.* [9] showed that the search strategy is optimal when the average duration of "motion phases" varies as the power either 3/5 or 2/3 of the average duration of "search phases," depending on the regime. The intermittent strategy, often referred to as "saltatory" [11], can be understood intuitively when the targets are difficult to detect and sparsely distributed. In this strategy the random walker presents two distinct phases: (1) a search phase, during which the searcher explores its immediate vicinity using its sensors. This scanning is modeled as a "slow" diffusive movement. (2) a motion phase, referred to during which the searcher moves "fast" and is unable to detect a target.

Bénichou *et al.* [9] assumed that the searcher randomly switches from phase 1 (respectively, 2) to phase 2 (respectively, 1) with a rate per unit time f_1 (respectively, f_2) and that the targets are immobile and randomly distributed with uniform density. The rates f_1 and f_2, which minimize the first passage time of the searcher at a target location, are given by two different regimes: First, if $f_{1max} \ll 1/\tau$, the optimal frequencies are such that $f_1 = f_{1max}$ and

$$f_2 = \left(\frac{4}{3\tau}\right)^{1/3} f_1^{2/3} \ . \tag{7}$$

Where $\tau = D/v^2$, D is the diffusion coefficient and the velocity v. In this regime (R1), the random walker spends more time searching than moving.

Second, if $f_{1max} \gg 1/\tau$ the optimal frequencies are such that $f_1 = f_{1max}$ and

$$f_2 = \left(\frac{2\sqrt{2}}{\tau}\right)^{1/3} f_1^{3/5} \ . \tag{8}$$

In this regime (R2), the random walker spends more time moving than searching. Figure 1d shows a plot of a simulated intermittent strategy using regime 1.

2.2 Systematic Searches

Bartumeus *et al.* [3] argue that systematic search strategies only work when some a priori relevant (although partial) information about targets (deterministic algorithms fixed and organized plans).

Using Sequences of Knots as a Random Search. Knots could be considered as a sequence of movements or steps based on a set of simple rules, these rules can be projected on a square lattice following the axes with consecutive steps with the aim to form a stable structure [12].

Fink *et al.* [12,13] suggest six states: $R_{\odot}, R_{\otimes}, C_{\odot}, C_{\otimes}, L_{\odot}$ and $L_{\otimes}$. These states may be considered as movements, hence, a consecutive array of well structured steps on a square lattice represents a knot. R for a right move, L for a left move and C for a centre move. A knot K_i is formed by a finite number of vertices joined by a group of links with the aim to fit a topological structure.

Pina-Garcia and D. Gu [14] define R and L as follows: if $\alpha_i \leq \tau \leq \beta_i$ then $p_r(t+1) = p_r(t) + v$ and the robot turns to the right or to the left an amount of $\theta(t)$ from the normal distribution $N(0, 90)$, where p_r is the robot position at time t, v is the speed, θ is a random number between 0 and 90 degrees and τ is the time step between two limits α_i and β_i. Subsequently, C is defined as a persistent direction with speed v. For implementation details see [14]. Figure 1e shows a plot of a simulated sequence of knots as a random search.

3 Composite Random Walk

In some cases when systematics search becomes less effective, the random walker must attempt to move in such a way so as to optimize their chances of locating targets by increasing the chances of covering a given area [3].

We propose a composite random walk that switches between a systematic strategy and a random search strategy. This switch is based on environmental changes (encounter rate) sensed by the robot, we use an adaptive switching behavior (biological fluctuation) defined by equations 1 to 6. Specifically, we compute $P(t) = \exp(-z(t))$ with a conditional function where if $P(t) = 1$, then a sequence of knots is triggered according to a deterministic algorithm with a finite time correlated to a decaying period of the environment activity $A(t)$. Otherwise, our random walker presents a ballistic motion as a default behavior defined as having $\mu \to 1$ from a Lévy walk (see algorithm 1).

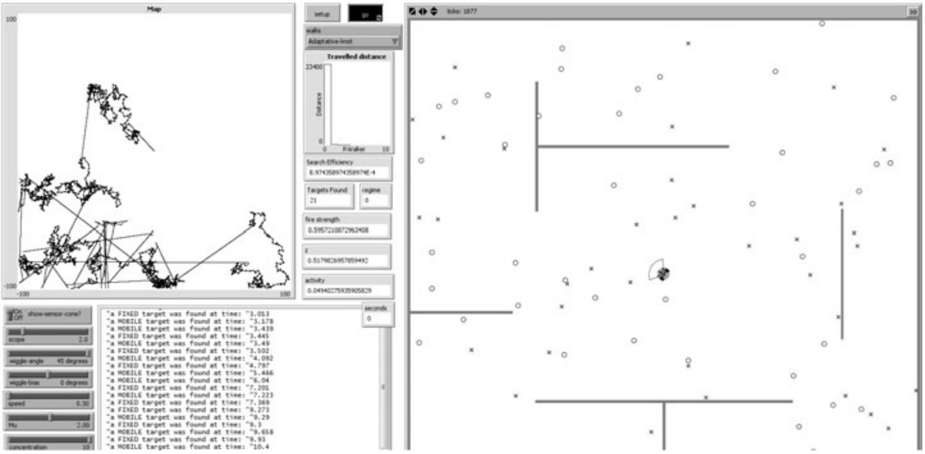
This composite random walk uses a stochastic model for interacting with a patchy environment. Thus, depending of the value of the activity $A(t)$, a strong tendency to switch behavior will be displayed by the robot. Figure 1f shows a plot of a simulated composite random walk.

Algorithm 1. Composite random walk that switches between a systematic strategy and a random search strategy

```
begin
    $A_{min} \leftarrow 0.0001$ // Minimum activity
    $C \leftarrow 0.999$ // $0 < C < 1$
    $K_F \leftarrow 0.1$ // A constant with a large value with respect to $A_{min}$
    $h \leftarrow 0.7$ // Position of the attractor
    $\epsilon(t) \leftarrow 0.5$ // White noise

    // Activity level for $\alpha(t)$
    $\alpha(t) \leftarrow (C * \alpha(t-1)) + (K_F * F(t))$
    $\alpha(t-1) \leftarrow \alpha(t)$

    // The fitness of the environment is defined by the activity parameter $A(t)$
    if $\alpha(t) \leq A_{min}$ then
        $A(t) \leftarrow A_{min}$
    else
        $A(t) \leftarrow \alpha(t)$

    $z(t) \leftarrow ((-2) * (z(t) - h) * A(t) + \epsilon(t))$ // Simple unimodal potential function

    // Switching probability at time t
    if $z(t) < 5$ then
        $P(t) \leftarrow \exp(-z(t))$

    // A sequence of knots or a ballistic motion is displayed according to $P(t)$
    if $P(t) \geq 1$ then
        $F(t) \leftarrow 0$
        Start sequence of knots // Systematic strategy
    else
        Start ballistic motion // Random search strategy
```

4 Simulations

We devised a group of simulations in order to evaluate search efficiency of six strategies (i.e., Brownian, Lévy, adaptive, intermittent, knots and composite). All the experiments were run and programmed on a java based platform (Netlogo [15]). A screen view of the simulator is depicted in Figure 2.



walks: Adaptative-knot

Targets Found: 21

regime: 0

show-sensor-cone?

scope: 2.0

wiggle-angle: 45 degrees

wiggle-bias: 0 degrees

speed: 0.30

Mu: 2.00

concentration: 10

seconds: 0

```
"a FIXED target was found at time: "3.013
"a MOBILE target was found at time: "3.178
"a MOBILE target was found at time: "3.439
"a FIXED target was found at time: "3.445
"a MOBILE target was found at time: "3.49
"a FIXED target was found at time: "3.502
"a MOBILE target was found at time: "4.092
"a FIXED target was found at time: "4.797
"a MOBILE target was found at time: "5.466
"a MOBILE target was found at time: "6.04
"a FIXED target was found at time: "7.201
"a MOBILE target was found at time: "7.223
"a FIXED target was found at time: "7.369
"a FIXED target was found at time: "9.273
"a MOBILE target was found at time: "9.29
"a FIXED target was found at time: "9.3
"a MOBILE target was found at time: "9.658
"a FIXED target was found at time: "9.93
"a MOBILE target was found at time: "10.4
```

Fig. 2. Our simulator: on your right side, showing a robot searching inside a complex environment. On your left side, plotting the path followed by the robot.

In all our experiments, targets were considered mobile and non-mobile in accordance with a destructive foraging [3]. The right side scenario depicted in Figure 2 was used for running all the experiments. Initial conditions were defined as follows: the size of the area is 100 × 100 (Netlogo units), initial position of the robot is located at (0,0) on the plane, velocity has been defined with a value of 0.30. In addition, 80 targets divided in two types were deployed: 40 non-mobile targets distributed randomly inside the environment and 40 mobile targets provided with a Brownian motion as a default behavior.

Search efficiency function S_{eff} is defined as follows

$$S_{eff} = \frac{\sum_{i=0}^{n} T_i}{d} \quad \text{where} \quad n \in \mathbb{N} \quad \text{and} \quad d \neq 0. \tag{9}$$

Where T_i denotes the number of targets that were found at time t and d is the traveled distance.

Simulations were tested using different parameter values. Specifically, parameters related to the biological switching behavior: $\epsilon = 0.5$, $h = 0.7$, $A_{min} = 0.0001$, $K_F = 0.1$, $C = 0.9999$ for adaptive switching behavior strategy and $C = 0.999$ for our composite random walk strategy. A destructive encounter dynamics was considered in the efficiency simulations, i.e. the target found by the random walker becomes undetectable in subsequent displacements (destructive foraging) [3].

5 Results

We performed 10 trials per each strategy with an average duration of 2000 seconds each one. Intermittent strategy was divided in two regimes $R1$ and $R2$ for experimental purposes. In every experiment traveled distance, search efficiency and targets found, were registered for subsequent comparisons. Thus, the mean and the standard deviation of each metric were computed to get the statistical support for evaluating the performance between every strategy. Our collected results are shown in table 1; from left to right we present name of the strategy, Lévy index μ, search efficiency, targets found and traveled distance.

Table 1. Performance of searching strategies. Ordered from left to right by name, Lévy index μ, search efficiency, targets found and traveled distance.

Name	μ	Search Efficiency	Targets Found	Traveled Distance
Brownian	$\mu = 3$	1.56×10^{-4}	38.8 ± 4.8	2.4×10^5
Lévy	$\mu = 2$	4.10×10^{-4}	79.3 ± 1.0	2.0×10^5
Adaptive	$\mu = 2, \mu = 3$	1.54×10^{-4}	46.8 ± 4.3	3.0×10^5
Intermittent R1	$\mu = 1$	1.87×10^{-4}	59.6 ± 3.9	3.1×10^5
Intermittent R2	$\mu = 1$	2.87×10^{-4}	72.1 ± 2.6	2.5×10^5
knots	N/A	3.18×10^{-4}	57.2 ± 7.2	1.7×10^5
Composite	$\mu = 1$	4.70×10^{-4}	80	1.7×10^5

The Lévy walk was used as a base model for deriving strategies such as: Brownian walk, adaptive switching behavior, intermittent strategy and our composite random walk (μ ranging between 1 and 3). It should be noted that, for the particular case of using sequences of knots as a random search (see [14]), this strategy is considered as a systematic search and uses a deterministic algorithm.

Search efficiency (SE) is plotted in Figure 3, where preliminary results suggest that our composite random walk is slightly more efficient than the rest of the strategies ($SE = 4.70 \times 10^{-4}$). In addition, for all cases the Lévy index plays a relevant role in the strategy; specifically, in the Lévy walk case ($\mu = 2$ and $SE = 4.10 \times 10^{-4}$) where long steps and less backtracking than Brownian random walk ($\mu = 3$ and $SE = 1.56 \times 10^{-4}$) is observed [1].

In contrast, using a ballistic motion ($\mu \to 1$) not necessarily implies a better efficiency as intermittent strategy (R1 $SE = 1.87 \times 10^{-4}$ and R2 $SE = 2.87 \times 10^{-4}$) shows, this is because in motion phases there is not any sensing action. However, the composite random walk combines a ballistic motion with a systematic search (sequences of knots), resulting in a significant improvement due to a super-diffusive behavior, i.e., covering as much ground as possible.

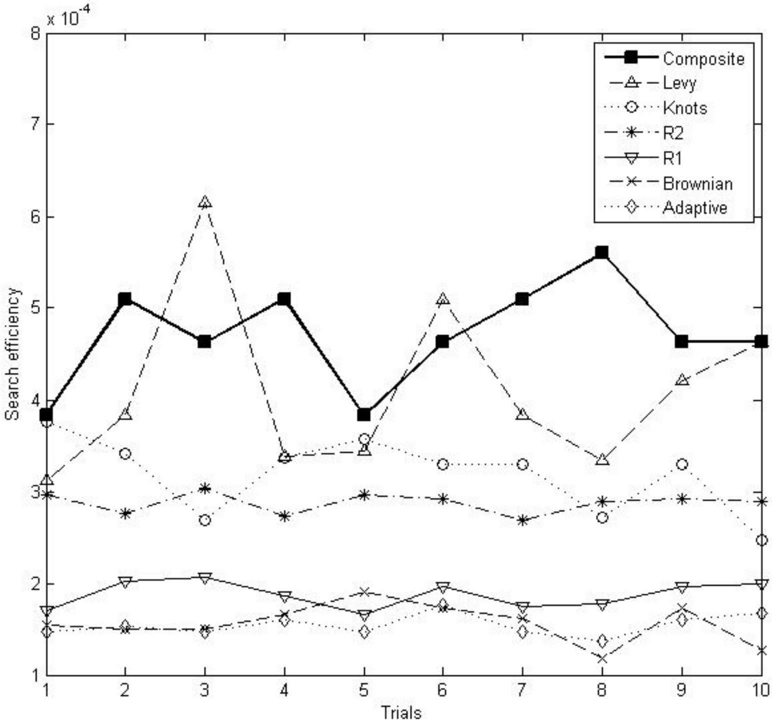


Fig. 3. Plot showing the search efficiency (*SE*) of six strategies: Brownian, Lévy, adaptive, intermittent (R1 and R2), knots and composite)

6 Conclusions and Future Work

Six different strategies were simulated, four random searches (Brownian, Lévy, adaptive and intermittent), one systematic search (sequences of knots) and our composite random walk. The resulting data obtained from experiments were analyzed by comparing the best search efficiency for each strategy.

Our results showed that combining a random search with a deterministic algorithm improved significantly a searching task where environmental uncertainty is unavoidable. In this study, we have proposed a composite random walk that switches from a failed strategy into a better strategy that optimizes encounter rates with the targets. In addition, our comparative results show that the search efficiency mainly depends on a scale invariant and super-diffusive phenomena. Thus, for each sort of environment, there might be a range of "original" strategies to speed up concrete search problems in robotics exploration.

Our results suggest that, a simple ballistic motion combined with a deterministic algorithm might optimize searching time of non-mobile and mobile targets, by filling up the search space uniformly.

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