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Leader-follower Flocking Experiments Using Estimated Flocking Center

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***Abstract*— We define a leader-follower flocking system in which a few members are group leaders who have global trajectory knowledge, while majority members are group followers who can communicate with neighbors, but do not have global trajectory information. The followers even do not know who are the leaders in the group. All group members estimate the position of flocking center by using a consensus algorithm via local communication in order to keep the flocking group connected. Based on the estimated positions of flocking center, a leader-follower flocking algorithm is proposed and its stability is proved. A group of real robots, "wifibots", are used to test the feasibility of the flocking algorithm. This leader-follower flocking system can track desired trajectories led by group leaders.**

***Index Terms*— multiple robot, consensus algorithm, distributed system.**

I. INTRODUCTION

For animals that forage or travel in groups, in many cases, a few individuals have global information, such as knowledge about the location of a food source, or a migration route. The informed individuals play an important role in guiding those that are less experienced [1]. For example, the foraging behavior of fish schools is known to be influenced by a few informed individuals and such a group can navigate towards a target [2]. Honeybee swarms can be guided to a new site by a few individuals [3]. For some animal species, the less experienced members even do not know which individuals, if any, have global information. Such a leader-follower flocking strategy can be used for mobile robots, where a few robots possess complicated sensory ability, such as GPS, and powerful computation ability to plan the trajectory. The followers simply use local sensors or limited communication to exchange information and control to flock without separation and collision.

In robotics, the leader-follower formation control was proposed in [4] [5]. It needs to build the dynamic model of distance and angle between leaders and followers. This means mobile robots know who are their leaders. In [6], a leader-follower system was investigated in terms of controllability and optimal control given the graph is connected and the followers know who are the leaders in the group. In [7], different leader roles were discussed and a convergent condition was constructed by using the contraction theory. The convergent condition needs to use global information. In [8], a leader based containment control strategy was designed. The group members clearly know which members are the leaders and the leaders have a desired formation pattern. The proposed centralized approach in [9] can be used for leader-follower flocking where the leader has a pre-defined trajectory to move and is independent of other members. All other members do not know who is the leader and control their motions to keep connected and avoid collision.

In the leader-follower flocking systems, the connectivity of flocking network is vital to flock as leaders might focus on tracking and followers might move away from flocking group. There are several approaches to handle the connectivity problem in general flocking systems, including a measure of local connectedness [10], a specific potential function [6] [11], distributed algorithm of eigenvector computation [12], and graph adjacency matrix [9].

If all members know a desired trajectory information, the cohesive force generated by this trajectory tracking force keeps the flocking group connected [15]. In the leader-follower flocking system considered in this paper, as only leaders have desired trajectory information, followers have the potential to move away from the flocking group and lead to group split. To solve this problem, we propose to estimate the position of flocking center by all group members. Due to the trajectory of flocking center being very close to the desired tracking trajectory, all members including the followers can use this knowledge to keep connected.

The accurate flocking center calculation requires a centralized approach. In this paper, we propose to use a consensus algorithm to estimate the position of flocking center. Although the robots still need to know their own position in global coordinate, only leader robots need to know desired trajectory information and flocking members to communicate with their neighbors. Therefore the flocking algorithm presented in this paper is still a distributed algorithm.

The experimental tests are conducted on a group of wifibots [16]. The wifibots are networked robots which can use wireless communication to exchange information with neighbors. In the proposed algorithm, although followers do

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not need to know the desired trajectory, all members need to know their positions. Our testing is carried out in an in-door environment (circular Robot Arena in University of Essex, around $100m^2$). There is a 3D tracking system (the Vicon tracking system) equipped in the lab with sub-millimeter accuracy. All wifibots can connect with the Vicon system via TCP/IP protocol and acquire their positions.

In the rest of this paper, section II introduces the structure of the flocking system and the estimation of flocking center. Section III presents the leader-follower flocking algorithm and stability proof. Section IV shows experimental results. Three real robots are used to test the algorithm. A brief conclusion is given in section V.

II. FLOCKING SYSTEM

We consider a flocking system consisted of N robots. The state of these N robots can be represented by a vector $q = [q_1^T, q_2^T, \cdots, q_N^T]^T \in \mathbf{R}^{2N}$, where $q_i = [x_i, y_i]$ is the position of robot i. The topology of flocking network can be represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ is the set of vertices (robots) and $\mathcal{E}$ is the set of edges (communication channels). The communication range of robots is denoted as C. The topology of flocking network depends on the distance between robots; that is a link only exists between a robot pair when their distance is smaller than C. When robot j is the neighbor of robot i, we have $j \in \mathcal{N}_i = \{j \in \mathcal{V}, j \neq i : ||q_i - q_j|| \leq C\}$. We assume that the communication range is the same for all the robots, so the graph of flocking network is undirected. For a connected graph, Laplacian matrix $\mathcal{L}$ is symmetric and positive semi-definite. Its minimum eigenvalue is 0 and the corresponding eigenvector is $\mathbf{1} = [1, \ldots, 1]^T$ or $\mathcal{L}\mathbf{1} = 0$ [13].

Three wifibots are used for flocking test. Actually the algorithm proposed in this paper is not limited to three robots. It can be applied to N robots. Each wifibot sends out its state q_i via wireless communication. Due to limited space used, all robots can receive states from all other robots. To simulate the limited range, a robot only receives states from neighbors who are located within the distance of C and discards states from other robots whose distances are larger than C. Each robot can contact with the tracking system (Vicon system) to obtain its own state q_i.

Fig. 1 is a diagram of the internal software structure of a robot. The core component is the controller which takes state information as input and outputs two velocities v, ω to move the robot. The robot model we used is kinematic model:

$$\begin{cases} \dot{x}_i^b = v_i \cos\theta_i \\ \dot{y}_i^b = v_i \sin\theta_i \\ \dot{\theta}_i = \omega_i \end{cases} \quad (1)$$

where θ_i is the heading of robot i and ($q_i^b = [x_i^b, y_i^b]^T$) represents the center point of wifibot. The robot state $q_i = [x_i, y_i]^T$ is referred to the hand position which is a point located at the heading axis with distance L to the center of robot. In the following, we will use q_i and its velocity $\dot{q}_i$ as robot state for our investigation. A coordination transformation can be made to obtain v_i, ω_i from $\dot{q}_i$.

$$\begin{aligned} v_i =& \dot{x}_i \cos\theta_i + \dot{y}_i \sin\theta_i \\ \omega_i =& \frac{1}{L}\Big[\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i\Big] \end{aligned} \quad (2)$$

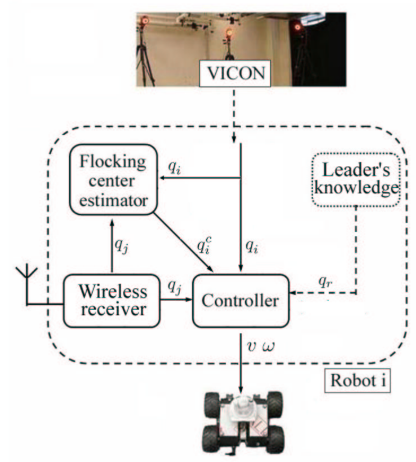

Fig. 1. Software structure

All other components in fig. 1 are related to state estimation, including predicting its own state q_i by using a jump Kalman filter, acquiring neighbor's state q_j via wireless communication, estimating the position of flocking center q_i^c, and obtaining the desired trajectory q_r (for leaders only). TCP/IP protocol is used to obtain state q_i from the Vicon system. We assume a full state observation without noise can be obtained.

The estimation of flocking center is a key element in our algorithm. It can be used for each robot to move close towards each other and avoid flocking split. Flocking robots can reach agreement on their aggregate information via consensus between adjacent robots. The consensus algorithm of robot i to estimate the position of flocking center is as follows:

$$\dot{q}_i^c = -\big[\sum_{j \in N_i} (q_i^c - q_j^c)\big] - q_i^c + q_i \quad (3)$$

where q_j^c is initialized as q_j. The vector form

$$\dot{q}^c = -\mathcal{L}q^c - q^c + q \quad (4)$$

where $q^c = [q_1^c, q_2^c, \ldots, q_N^c]^T$.

q_i^c should asymptotically converge to the average of coordinates q_i:

$$q_i^c \rightarrow \frac{1}{N}\sum_{i=1}^{N} q_i \quad (5)$$

We can show that q_i^c in the consensus algorithm can be global asymptotically ε stable to the average of coordinates $\frac{1}{N}\mathbf{1}\mathbf{1}^T q$.

Defining the error between q^c and the average of inputs $\frac{1}{N}\mathbf{1}\mathbf{1}^T q$ of the consensus filter as e:

$$e = \frac{1}{N}\mathbf{1}\mathbf{1}^T q - q^c \tag{6}$$

Because $\mathbf{1} = [1, \ldots, 1]^T$, $\mathbf{1}\mathbf{1}^T$ is a $N \times N$ matrix in which all the elements are 1. The derivative of the error is given by

$$\begin{aligned} \dot{e} &= \frac{1}{N}\mathbf{1}\mathbf{1}^T \dot{q} - \dot{q}^c \\ &= \frac{1}{N}\mathbf{1}\mathbf{1}^T \dot{q} + \mathcal{L}q^c - q + q^c \\ &= -(\mathcal{L} + I)e + \frac{1}{N}\mathbf{1}\mathbf{1}^T \dot{q} + \frac{1}{N}\mathbf{1}\mathbf{1}^T q - q \end{aligned} \tag{7}$$

where we use $\mathcal{L}\frac{1}{N}\mathbf{1}\mathbf{1}^T q = 0$ in a connected graph. The ith element of the vector

$$\tfrac{1}{N}\mathbf{1}\mathbf{1}^T \dot{q} + \tfrac{1}{N}1\mathbf{1}^T q - q$$

can be expressed as:

$$\tfrac{1}{N}\textstyle\sum_{j=1}^N \dot{q}_j + \tfrac{1}{N}\sum_{j=1}^N (q_j - q_i)$$

Assuming $\dot{q}_i$ and q_i are uniformly bounded, i.e. the robot position and the rate of the robot position are uniformly bounded. We have

$$\|\frac{1}{N}\sum_{j=1}^N \dot{q}_j + \frac{1}{N}\sum_{j=1}^N (q_j - q_i)\| \leq c \tag{8}$$

where c is a positive constant.

A positive definite function $V = \frac{1}{2}e^T e$ can be used to prove the ε stability. Due to $e^T(\mathcal{L}+I)e \geq \lambda_{min}(\mathcal{L}+I)\|e\|^2 = \|e\|^2$ in a connected graph [13], we have

$$\begin{aligned} \dot{V} &= -e^T(\mathcal{L} + I)e + e^T(\frac{1}{N}\mathbf{1}\mathbf{1}^T \dot{q} + \frac{1}{N}\mathbf{1}\mathbf{1}^T q - q) \\ &\leq -\|e\|^2 + e^T c\mathbf{1} \\ &\leq -\|e\|^2 + c\sqrt{N}\|e\| \end{aligned} \tag{9}$$

where Jensen's inequity $e^T c\mathbf{1} \leq c\sqrt{N}\|e\|$ is used. Let

$$\varepsilon = c\sqrt{N} \tag{10}$$

We have

$$\dot{V} \quad \leq \quad -\left(\|e\| - \frac{\varepsilon}{2}\right)^2 + \left(\frac{\varepsilon}{2}\right)^2 \tag{11}$$

By defining the set $\Omega_c = \{e : V(e) \leq \varepsilon^2/2\}$ and using LaSalle invariance principle, it can be seen that any error e, which is not in Ω_c, will move into Ω_c because of $\dot{V} < 0$ and it will remain in Ω_c. Thus the error is global asymptotically ε stable.

III. FLOCKING ALGORITHMS AND STABILITY

The leader-follower flocking system in this paper is composed of minority leaders and majority followers. The leaders have knowledge of a desired trajectory and need to track this trajectory. At the same time, they also need to avoid collision and move to the flocking center. Eventually, the leaders can lead the entire flocking group to track the desired trajectory. The followers do not know who are the leaders in the group and do not know the desired trajectory. They only need to avoid collision and move to the flocking center.

The position estimation of flocking center is the same to both leaders and followers. Avoiding collision is another common point to them. Simply, adjacent robots should keep a specific distance. If the distance between adjacent robots is too small, they attempt to separate. There are several approaches to design a separation potential function $H_s(d_{ij})$ where $d_{ij} = ||q_i - q_j||$. We have used fuzzy logic to design $H_s(d_{ij})$. It is continuous function and its gradient $\nabla H_s(d_{ij})$ owns the following properties:

- When the distance (d_{ij}) between robots i and j is smaller than a specific distance d, $\nabla H_s(d_{ij})$ is negative. Robot i moves away from j.
- When the distance (d_{ij}) between robots i and j is larger than the specific distance d, $\nabla H_s(d_{ij})$ is close to zero and equal to zero when $d_{ij} = d$.

Fig. 2 illustrates an example of $f_s(d_{ij})$ designed by using fuzzy logic.

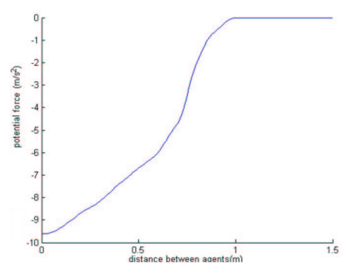

Fig. 2. Separation force function $f_s(d_{ij})$

To simplify the analysis, we consider the flocking system with only one leader l. The leader flocking algorithm is designed as follows:

$$\dot{q}_l = -\,k_c(q_l - q_l^c) - \sum_{j\in\mathcal{N}_l} \nabla H_s(d_{lj}) - k_r(q_l - q_r) \tag{12}$$

where k_c and k_r are positive gains and q_r is the position of the desired trajectory.

We define the set of all the followers as $F = \{\ i \mid i \neq l,\ i \in \mathcal{V}\}$. The follower flocking algorithm is designed as follows:

$$\dot{q}_i = -\,k_c(q_i - q_i^c) - \sum_{j\in\mathcal{N}_i} \nabla H_s(d_{ij}) \quad (i \in F) \tag{13}$$

In the following, we analyze the stability of the above algorithm by using LaSalle's invariance principle. Assuming the position estimation of flocking center for each robots reaches its true value $q_1^c = \cdots = q_N^c = q^c = 1/N \sum_{i=1}^{N} q_i$.

Let $x_i = q_i - q_r$ and $x^c = q^c - q_r$. The leader-follower flocking algorithm can be written as:

$$\begin{aligned} \dot{x}_l =& - k_c(x_l - x^c) - \sum_{j \in \mathcal{N}_l} \nabla H_s(d_{lj}) - k_r(x_l - x_r) \\ \dot{x}_i =& - k_c(x_i - x^c) - \sum_{j \in \mathcal{N}_i} \nabla H_s(d_{ij}) \quad (i \in F) \end{aligned} \tag{14}$$

Let $H(x_i)$ denote the Hamiltonian function of robot i and the Hamiltonian function of the flocking system is:

$$H(x) = \sum_{i=1}^{N} H(x_i) \tag{15}$$

Define the Hamiltonian functions of leaders and followers as follows:

$$\begin{aligned} H(x_l) =& \frac{1}{2} k_c ||x_l - x^c||^2 + \sum_{j \in \mathcal{N}_l} H_s(d_{lj}) + \frac{1}{2} k_r ||x_l - x_r||^2 \\ H(x_i) =& \frac{1}{2} k_c ||x_i - x^c||^2 + \sum_{j \in \mathcal{N}_i} H_s(d_{ij}) \quad (i \in F) \end{aligned} \tag{16}$$

So

$$\dot{H}(x) = \sum_{i=1}^{N} \nabla H(x_i) \cdot \dot{x}_i = - \sum_{i=1}^{N} \dot{x}_i^2 \tag{17}$$

which means $\dot{H}(x)$ is non-positive. According to LaSalle's invariance principle, we know all the robots attempt to approach to the stable state:

$$\dot{x}_1 = \dot{x}_2 = \cdots = \dot{x}_n = 0 \tag{18}$$

Because of $\dot{x}_i = \dot{q}_i - \dot{q}_r$, the robot velocities equal to the velocity of desired trajectory at the stable state.

IV. EXPERIMENTS

The experimental tests are conducted in the Robot Arena at University of Essex. The Vicon system is used to provide position information to each of wifibots. We use TCP/IP protocol to implement the communication.

In the experiment, it is assumed that there exists only local neighbor-to-neighbor information exchange among the robots and the robots can only receive their own states from the Vicon system. The maximum velocity of robot is $2000mm/s$. The desired trajectory is centered at $[0, 0]$ with a radius of $1700mm$ and the leader robot knows this trajectory:

$$\begin{aligned} x_r &= 1700 \cos 0.05t \\ y_r &= 1700 \sin 0.05t \end{aligned}$$

Fig. 3 shows the trajectories of the first flocking experiment. The solid line denotes the trajectory of the leader; the dashed lines are the trajectories of the two followers. Generally, it can be seen that the leader always attempts to stay in the front of the flock, and the whole group can tracking the desired trajectory.

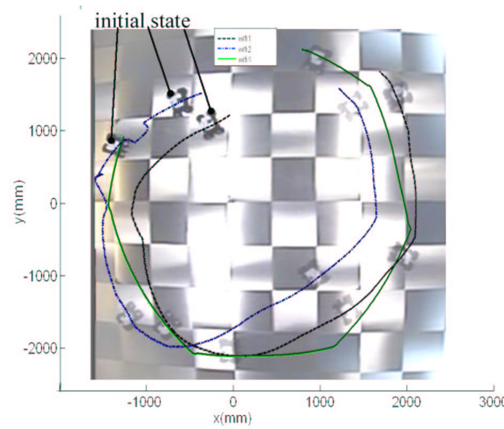

Fig. 3. The first flocking experiment

Fig. 4 shows the distances between robots. The expected distance between robots is set as $d = 600mm$. Initial distances between any two robots are quite large. Gradually they converge to the specific value and keep stable at a reasonable level. Fig. 5 shows the cohesion radius, which is the maximum radius of the flocking group. It changes from large value at the initial place to small values, which means the flocking is moving to be cohesive.

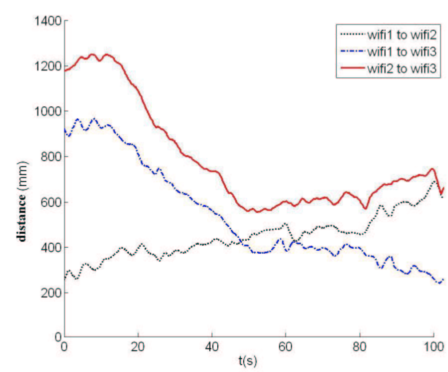

Fig. 4. Distance between robots

In the above experiment, only one loop is used to estimate the position of flocking center by using the consensus algorithm, i.e. the robots only communicate with neighbors for once at each step. The outcomes of the estimation of flocking center are recorded by each of the robots during the flocking. We compare the estimated centers with the "actual flocking center" calculated by using the data from the Vicon system. Fig. 6 illustrates three estimated flocking centers and

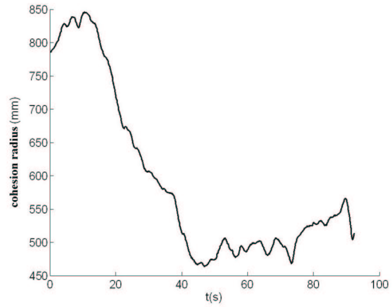

Fig. 5. Cohesion radius

an actual flocking center. The results show although only one loop is used in the consensus algorithm, the estimated results are very accurate. We have already proven the flocking algorithm is stable even if one loop is used in our another paper. The differences between them are also calculated and plotted in fig. 7. It can be seen that the differences gradually decrease down to about $50mm$.

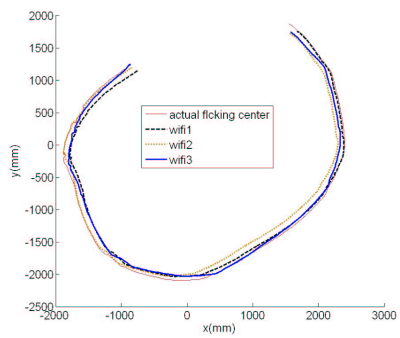

Fig. 6. Estimate flocking center and actual flocking center

In the second experiment, we place an obstacle on the desired trajectory to show how the flocking group avoids obstacle. The obstacle position is provided to each robot by the Vicon system. No change is made to the flocking algorithm. Both leader and followers treat the obstacle as a flocking member and avoid it through the separation function of the algorithm. Wifibot1 is placed on the front of the group and only has one neighbor (wifibot2). Wifibot2 is placed in the middle of the group and has two neighbors. Wifibot3 is placed on the back of the group and has one neighbor (wifibot2).

Fig. 8 illustrates the trajectories of the flocking. The solid line denotes the trajectory of wifibot1. It shows that wifibot1 moves back at the beginning of the flocking as it wants to move close to the estimated flocking center. The middle robot (wifibot2) is the leader. Gradually three robots move from a line pattern to a triangle pattern.

The obstacle is marked with a blue circle on the top of the

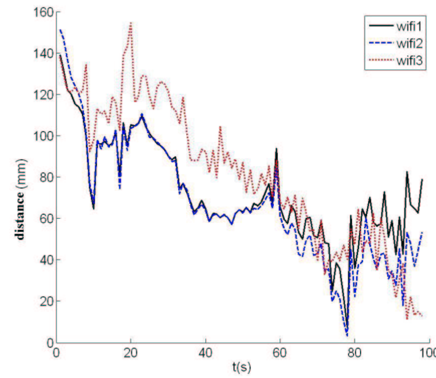

Fig. 7. Differences between estimated flocking center and actual flocking center

figure. It can be seen that the flocking smoothly avoids the obstacle and regroups together after passing the obstacle.

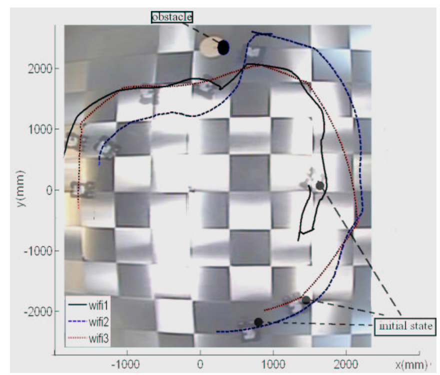

Fig. 8. The second flocking experiment with obstacle

Fig. 9 illustrates the distances between the robots. Initially, the distance between wifibot1 and wifibot2 is quite large. At $t = 12s$, wifibot1 joins the group and forms a triangle pattern. After that, the distances decrease to a specific level. When the flocking group encounters the obstacle at about $t = 50s$, the distances increase due to the maneuver of obstacle avoiding. However they are still connected as a group. After passing the obstacle, the distances decrease and the group tracks the desired trajectory with a triangle pattern again. The cohesion radius shown in fig. 10 has a similar change.

In the second experiment, the consensus algorithm runs five loops at each step to reduce the estimated error of flocking center. The differences between the three estimated positions of flocking center and the actual flocking center are shown in fig. 11. The three estimated results are very close to each other. At the beginning of the flocking, all the estimated errors are quite large as they are placed far from each other and the initial estimated positions of flocking center are initialized with their initial positions. When they move close enough (from a line pattern to a triangle pattern), the estimated results become more accurate. The fluctuation at about $t = 50s$ is caused by the behavior of obstacle avoiding.

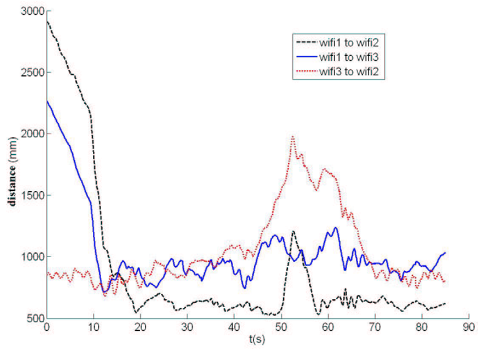

Fig. 9. Distances between the robots

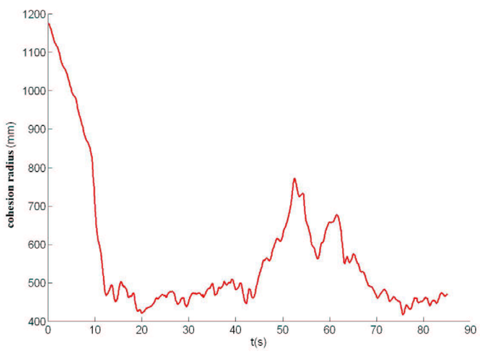

Fig. 10. Cohesion radius

V. CONCLUSIONS

Some biological flocking systems have shown that a leader-follower flocking group consists of a few individuals who have global information and other members who even do not know which individuals, if any, have such information. The informed individuals play an important role in guiding those that are less experienced. This paper proves such a leader-follower flocking system is stable and demonstrates the experiments of such a leader-follower flocking system using real robots.

One of important elements in the leader-follower flocking algorithm is the position estimation of flocking center. We employ a consensus algorithm to achieve the distributed estimation. The robots only use local neighbor-to-neighbor communication for flocking. The information about flocking center can keep flocking group connected.

The experiments successfully show the proposed algorithms work as expected. The algorithms also have the ability to avoiding obstacle.

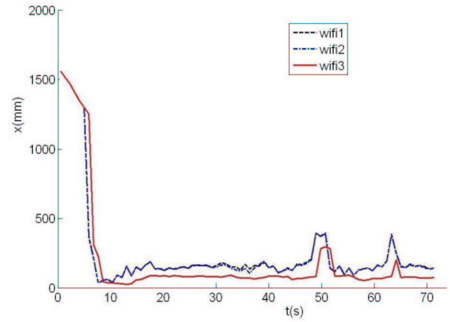

Fig. 11. Differences between estimated flocking center and actual flocking center

Our further work will focus on the investigation of connectivity problem of this flocking algorithm. We are proving the group can keep connected by using the flocking center information.

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