

Modeling and stability analysis of grey–fuzzy predictive control

Lisheng Wei^a, Minrui Fei^{a,*}, Huosheng Hu^b

^a Shanghai Key Laboratory of Power Station Automation Technology, School of Mechatronics and Automation, Shanghai University, Shanghai 200072, China

^b Department of Computing and Electronic Systems, University of Essex, Colchester CO4 3SQ, UK

ARTICLE INFO

Available online 3 September 2008

Keywords:

Grey prediction
Accumulated generating operation
Fuzzy control
Membership function
Switching mechanism
Networked control systems

ABSTRACT

This paper presents a grey–fuzzy predictive controller that is based on fuzzy theory, grey prediction and on-line switching algorithms. The grey predictor is applied to extract key information and reduce the randomness of the measured non-stationary time-series signals from sensors, and send the prediction information to the fuzzy controller. The complete mathematical model is derived and the sufficient condition for convergence is given. To achieve better transient performance and steady-state responses, an on-line switching mechanism is adopted to regulate appropriately the forecasting step size of the grey predictor, according to the error feedback from different periods of the system response. Experimental results obtained from a plant show that the control accuracy and robustness are much improved when the proposed new method is applied.

© 2008 Elsevier B.V. All rights reserved.

1. Introduction

Traditional fuzzy control (TFC) theory and methods have been widely used in industry, and the analysis of control rules and membership function parameters has been investigated extensively [14,15]. In general, the TFC strategy adopts the previously measured information, and the control signal is a function of the previous system error and its deviation, namely “delayed control”. In many circumstances, it is feasible and practicable [3,13–16], and guarantees the basic requirements of global stability and acceptable performance [15]. However, due to the system and sensor noise and the limitation of our cognitive abilities, the information and control rules we obtained are usually uncertain and limited in scope. This could affect the real-time performance and adaptability of fuzzy control systems, especially for networked control systems (NCSs) that contain inherent uncertainties such as package loss and time delay.

Since the birth of grey theory in 1982 [4,5], grey prediction has been applied to fuzzy control applications [2,12,20]. Cheng proposed a grey prediction controller to control an industrial process without the system model in 1986 [2]. Luo developed grey–fuzzy control algorithms for an autonomous mobile system, which can deal with the uncertain environmental conditions to ensure the tracking performance [12]. Yu designed a grey–fuzzy

controller in which a grey predictor is used to improve the speed response of a brushless DC motor [20].

The reason for using grey prediction in a fuzzy control system is that the grey predicted output from an unknown plant could always provide us some useful information for better control of the system before the system behavior runs into bad situations. However, this traditional grey predictive controller structure uses a fixed forecasting step size, and the grey predictor is always manipulated in the overall control process. The short-term oscillation occurs before the system response changing into steady states. When the response has not yet reached the set point, the output has begun to reduce and go down, which is not the desired performance.

Recently, many researchers focus on how to select a proper and dynamic forecasting step to control a system [1,6,9,18]. Different techniques have been adopted to regulate the grey forecasting step size, including fuzzy logic, neural networks, genetic algorithms, etc. From the experiments, we know that a grey predictor with a negative and fixed forecasting step size always has a small settling time and a large overshoot. On the other hand, a grey predictor with a large positive and fixed forecasting step size has a small overshoot and a large settling time.

To improve the fuzzy system's performance, we propose a novel grey–fuzzy predictive control (GFPC) strategy using an on-line dynamic switching mechanism. It finds a suitable forecasting step size for each control action of three modes, namely a big positive-step forecasting mode, a small positive-step forecasting mode and a negative-step forecasting mode. When the system error is large, the negative-step forecasting mode is used to increase the upward momentum of the output curve. This is to speed-up the system response for shortening the settling time.

* Corresponding author. Shanghai Key Laboratory of Power Station Automation Technology, Shanghai University, 149 Yanchang Road, Zhabei District, Shanghai 200072, PR China. Tel.: +86 21 5633 1261 807.

E-mail addresses: lishwei_11@163.com (L. Wei), mrfei@staff.shu.edu.cn (M. Fei), hhu@essex.ac.uk (H. Hu).

When the system error is small, the big positive-step forecasting mode is used to prevent the overshoot. The last condition is used when the middle error occurs.

For convenience, the fuzzy controller in this paper adopts traditional two-input and one-output structure which behaves approximately like a PD controller with variable parameters [14,19]. The entire input signal is obtained from the grey predictor according to the complementary behavior of the distinct modes. Then the sufficient condition for convergence is derived. From experimental results, we find this design not only can drastically reduce the system overshoot, but also can maintain the characteristic of the shorter settling time of the system compared to TFC.

The rest of this paper is organized as follows. In Section 2, the structure and the mathematical model of the GFPC is presented. The sufficient condition for convergence of the proposed control algorithm is derived. In Section 3, simulation results with the proposed control scheme are obtained. Finally, conclusion remarks and future work are presented in Section 4.

2. Design of grey-fuzzy predictive control

2.1. Traditional grey prediction model

The grey predictive method has been successfully used to model the dynamic systems in different fields such as agriculture, ecology, economy, statistics, meteorology, industry, environment, and so on [1,2,4–6,9,12,18,20]. It can predict poor, incomplete or uncertain messages in a system without the need of a long-term historical data. It reveals underlying regular conditions within a random time sequence via a special data processing.

Different from the existing statistic methods for prediction, grey predictive method uses data generation method, such as ratio checking (RC) and accumulated generating operation (AGO) to reduce the stochastic of raw datum and obtain more regular sequence from the existing information. The general form of a grey differential model is GM(*i,j*), where *i* is the order of the ordinary differential equation and *j* the number of grey variables. In general, the computing time of the grey predictive model increases exponentially as *i* and *j* increase. But prediction accuracy may not improve with large *i* or *j* values. Therefore, the traditional grey prediction model GM(1,1) is used in this paper, which can be derived by the following basic steps [2,4,5,12,20]:

Step 1—RC and AGO

By checking the plant output sequences ratio $Y^{(0)} = (y^{(0)}(1), y^{(0)}(2), \dots, y^{(0)}(n))$ and transforming the original sequences into new sequences with AGO, we have

$$RC: \quad \sigma^{(0)}(k) = \frac{y^{(0)}(k-1)}{y^{(0)}(k)} \quad (1)$$

$$AGO: \quad y^{(1)}(k) = \sum_{m=1}^k y^{(0)}(m) \quad (2)$$

where $k = 2, \dots, n$ is the sequences number.

The overlay area of new ratio sequences is defined as $(e^{-2/(n+1)}, e^{2/(n+1)})$. When the ratio sequences $\sigma^{(0)}(k)$ are in overlay area, we can transform the original sequences into new smooth sequences $Y^{(1)} = (y^{(1)}(1), y^{(1)}(2), \dots, y^{(1)}(n))$ with Eq. (2). Otherwise, some pre-treatment is required, including logarithmic transformation, root-squaring transformation, or translational method:

Step 2—Build grey model GM(1,1)

The GM(1,1) model can be constructed by establishing a first-order differential equation as follows:

$$y^{(0)}(k) + az^{(1)}(k) = b \quad (3)$$

where *a* and *b* are estimation parameters. $Z^{(1)}$ is affected by $Y^{(1)}$, i.e.

$$z^{(1)}(k) = 0.5y^{(1)}(k) + 0.5y^{(1)}(k-1) \quad (4)$$

Here, we set

$$Y_N = [y^{(0)}(2) \quad y^{(0)}(3) \quad \dots \quad y^{(0)}(n)]^T$$

$$B = \begin{bmatrix} -z^{(1)}(2) & -z^{(1)}(3) & \dots & -z^{(1)}(n) \\ 1 & 1 & \dots & 1 \end{bmatrix}^T, \quad M = \begin{bmatrix} a \\ b \end{bmatrix} \quad (5)$$

So, Eq. (3) can be substituted as

$$Y_N = BM \quad (6)$$

Then the optimal parameter *M* can be obtained by using the minimum least-square estimation algorithm

$$M = (B^T B)^{-1} B^T Y_N \quad (7)$$

According to the first-order differential equation, the grey model GM(1,1) becomes

$$y^{(0)}(k) + 0.5a[y^{(1)}(k) + y^{(1)}(k-1)] = b \quad (8)$$

That is,

$$(1 + 0.5a)y^{(0)}(k) + ay^{(1)}(k-1) = b \quad (9)$$

When $k > 2$, we have

$$y^{(0)}(k) = \frac{b - ay^{(1)}(k-1)}{1 + 0.5a} = \left(\frac{1 - 0.5a}{1 + 0.5a} \right) y^{(0)}(k-1) \quad (10)$$

$$y^{(0)}(k) = \left(\frac{1 - 0.5a}{1 + 0.5a} \right)^m y^{(0)}(k-m) \quad (11)$$

When $k = 2$, we have

$$y^{(0)}(2) = \frac{b - ay^{(0)}(1)}{1 + 0.5a} \quad (12)$$

Based on the above derivation, we obtain the grey prediction model as follows:

$$y_p^* = \left(\frac{1 - 0.5a}{1 + 0.5a} \right)^{(n+p-2)} \frac{b - ay^{(0)}(1)}{1 + 0.5a} \quad (13)$$

where *p* is the prediction step; y_p^* is the prediction value sent to the fuzzy controller. In this paper, the sequence number is set $n = 4$. Then the grey prediction model can be described as follows:

$$y_p^* = \left(\frac{1 - 0.5a}{1 + 0.5a} \right)^{(2+p)} \frac{b - ay^{(0)}(1)}{1 + 0.5a} \quad (14)$$

2.2. Modeling of grey-fuzzy predictive control

The configuration of the proposed GFPC with an on-line dynamic switching mechanism is shown in Fig. 1 in which $r(t)$ is the reference value, $y(t)$ is the sensor output value, $y^*(t) = y_p^*$ is the grey prediction value, $u(t)$ is the output of the fuzzy controller, and $e^*(t)$ is the deviation: $e^*(t) = r(t) - y^*(t)$. The whole control strategy is based on the prediction value $y^*(t)$ of the system output $y(t)$, $y^*(t)$ and $r(t)$ are transmitted to the fuzzy controller and a control signal $u(t)$ is generated to control the plant.

Since the forecasting step size decides the predictive value and finally affects the control performance, an on-line switching mechanism is adopted to regulate the appropriate forecasting step size of the grey predictor. In general, when the system error is large, the system response should be quick and the switching mechanism should choose the negative-step forecasting mode. Then, the grey predictor has the ability of predicting the “previous” behavior of the system, and the predictive value of

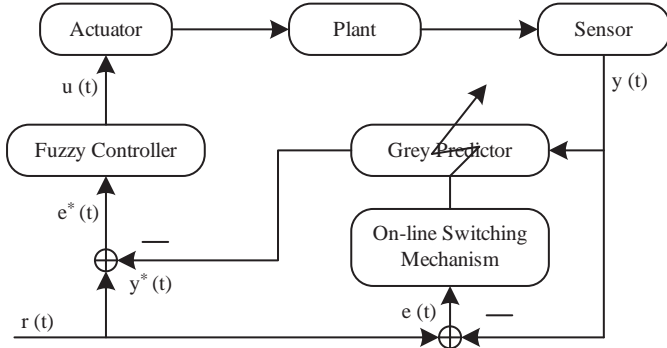


Fig. 1. The structure of grey-fuzzy predictive controller.

the output will be large for a decreasing system response. Therefore, the fuzzy controller will transmit a big control signal to speed-up the system response and result in a shorter settling time.

When the system error is very small, the system response should be decreased and the switching mechanism should choose the big positive-step forecasting mode. Then, the forecasting value of the output will be small so that the fuzzy controller generates a smaller forecasting control signal to prevent the system overshoot. Therefore, this mode leads to a slow system response so that the overshoot of the fuzzy control system with a grey predictor is smaller than that of the fuzzy control system without a grey predictor. However, this causes a long settling time. When the system error is in a special definite range, the switching mechanism should choose the small positive-step forecasting mode to overcome the drawback of the other two modes. Therefore, we have the following switching mechanism:

$$p = \begin{cases} p_1 < 0 & \text{if } e(t) > e_1 \\ p_2 > 0 & \text{if } e_s < e(t) < e_1 \\ p_3 > 0 & \text{if } e(t) < e_s \end{cases} \quad (15)$$

where p is the prediction step; p_1, p_2 and p_3 are the step sizes for the large error, the middle error and the small error, respectively; e_s and e_1 are the switching values of the small error and the large error, respectively.

Then the input variables of the fuzzy controller are

$$\text{input1: } e^*(t) = r(t) - y^*(t)$$

$$\text{input2: } \Delta e^*(t) = \dot{r}(t) - \dot{y}^*(t) = [e^*(t) - e^*(t - T)]/T$$

When the structure of the fuzzy controller uses the linear control rules and Mamdani minimum inference, the fuzzy control strategy behaves approximately like a variable parameter PD controller [14]. Then the GFPC algorithm has

$$u(t) = u(t - T) + k_1 e^*(t) + k_2 \Delta e^*(t) \quad (16)$$

where k_1 and k_2 are the variable parameters.

2.3. Stability analysis of grey-fuzzy predictive control

Let us consider a linear model described as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (17)$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^r$ are the states, control input, and output of the system, respectively. $A, B,$ and C are real matrices of appropriate dimensions. $e(t)$ is the tracking error defined as

$$e(t) = r(t) - y(t) \quad (18)$$

Then we have

$$\begin{aligned} e^*(t) &= r(t) - y^*(t) \\ \Delta e^*(t) &= \dot{e}^*(t) = \dot{r}(t) - \dot{y}^*(t) \end{aligned} \quad (19)$$

In Eq. (19), $y^*(t)$ is defined as the output of the predictive system using the grey prediction method. The effect of uncertainty in the prediction will be compensated if the actual error is available in the next iteration.

According to Eq. (14), we can make assumption that prediction error is also bounded to a certain small positive constant b_p such that

$$\|y^*(t) - y(t)\|_i \leq b_p \quad (20)$$

$$\|e^*(t) - e(t)\|_i \leq b_p \quad (21)$$

Prediction error bound b_p is a measure to represent the deviation of $e^*(t)$ from $e(t)$, which means that the higher b_p value, the poorer the grey prediction of the system.

Let

$$y(t) = Cx(t) = C \left[\Phi(t)x_0 + \int_0^t \Phi(t - \tau)Bu(\tau) d\tau \right] \quad (22)$$

According to Eq. (16), we have

$$e(t) = r(t) - y(t) = r(t) - C \left[\Phi(t)x_0 + \int_0^t \Phi(t - \tau)Bu(\tau) d\tau \right] \quad (23)$$

Putting the GFPC algorithm Eqs. (16)–(23), assuming $r(t)$ is constant, x_0 and $e^*(0)$ are equal to zero, we have

$$\begin{aligned} e(t) &= r(t) - C\Phi(t)x_0 - \int_0^t C\Phi(t - \tau)Bu(\tau - T) d\tau \\ &\quad - \int_0^t C\Phi(t - \tau)B[k_1 e^*(\tau) + k_2 \dot{e}^*(\tau)] d\tau \\ &= e(t - T) - \int_0^t H(t - \tau)[k_1 e^*(\tau) + k_2 \dot{e}^*(\tau)] d\tau \end{aligned} \quad (24)$$

where $H(t - \tau) = C\Phi(t - \tau)B$.

$$\int_0^t H(t - \tau)k_2 \dot{e}^*(\tau) d\tau = H(0)k_2 e^*(t) - \int_0^t \frac{\partial}{\partial \tau} [H(t - \tau)k_2] e^*(\tau) d\tau \quad (25)$$

Now, using Eq. (25) to simplify and rearrange (24), we have

$$\begin{aligned} e(t) &= e(t - T) - \int_0^t H(t - \tau)[k_1 e^*(\tau) + k_2 \dot{e}^*(\tau)] d\tau \\ &= e(t - T) - \int_0^t H(t - \tau)k_1 e^*(\tau) d\tau \\ &\quad - H(0)k_2 e^*(t) + \int_0^t \frac{\partial}{\partial \tau} [H(t - \tau)k_2] e^*(\tau) d\tau \\ &= e(t - T) - H(0)k_2 e^*(t) \\ &\quad - \int_0^t \left\{ H(t - \tau)k_1 - \frac{\partial}{\partial \tau} [H(t - \tau)k_2] \right\} e^*(\tau) d\tau \end{aligned} \quad (26)$$

For brevity of our discussion, the notation and norms of the functions, introduced in this paper, are as follows:

$$\begin{aligned} \|f(t)\|_i &= \sup e^{-\lambda t} \|f(t)\| \\ \|f(t)\|_\infty &= \sup \|f(t)\| \\ b &= \sup \left\| H(t - \tau)k_1 - \frac{\partial}{\partial \tau} [H(t - \tau)k_2] \right\|_\infty \end{aligned} \quad (27)$$

Taking the norm $\|\cdot\|$ both sides of Eq. (26), we have

$$\begin{aligned} \|I + H(0)k_2\| \|e(t)\| &\leq \|e(t - T)\| + \|H(0)k_2\| \|e^*(t) - e(t)\| \\ &+ \int_0^t \left\| H(t - \tau)k_1 - \frac{\partial}{\partial \tau} [H(t - \tau)k_2] \right\| \|e^*(\tau) - e(\tau)\| d\tau \\ &+ \int_0^t \left\| H(t - \tau)k_1 - \frac{\partial}{\partial \tau} [H(t - \tau)k_2] \right\| \|e(\tau)\| d\tau \\ &\leq \|e(t - T)\| + \|H(0)k_2\| \|e^*(t) - e(t)\| \\ &+ \int_0^t b \|e^*(\tau) - e(\tau)\| d\tau + \int_0^t b \|e(\tau)\| d\tau \end{aligned} \quad (28)$$

Then, multiplying both sides of Eq. (28) by $e^{-\lambda t}$, using expression (20) and assuming that λ is taken to be sufficiently large, we obtain

$$\begin{aligned} \|I + H(0)k_2\| \|e(t)\|_\lambda &\leq \|e(t - T)\|_\lambda + \|H(0)k_2\| b_p \\ &+ b \frac{1 - e^{-\lambda t}}{\lambda} b_p + b \frac{1 - e^{-\lambda t}}{\lambda} \|e(t)\|_\lambda \end{aligned} \quad (29)$$

Rearranging Eq. (29), we have

$$\|e(t)\|_\lambda \leq \gamma_1(t) \|e(t - T)\|_\lambda + \gamma_1(t) \gamma_2(t) b_p \quad (30)$$

where

$$\begin{aligned} \gamma_1(t) &= \left[\|I + H(0)k_2\| - b \frac{1 - e^{-\lambda t}}{\lambda} \right]^{-1} \\ \gamma_2(t) &= \|H(0)k_2\| + b \frac{1 - e^{-\lambda t}}{\lambda} \end{aligned}$$

Then we have the following theorem.

Theorem. *If the GFPC, whose structure uses the linear control rules and Mamdani minimum inference, is applied to the system (17) with the beneath mentioned assumptions and condition, such that $\gamma_1(t) < 1$, then the tracking error $\|e(t)\|_\lambda$ satisfies Eq. (30) for sufficiently large λ .*

According to Eq. (30), the absolute convergence of error is not guaranteed because of the presence of b_p that is the bound of the prediction error. Therefore, in the presence of uncertainties, the prediction error may not converge to zero but to a certain bound.

If the prediction is very accurate, which means that the parameter identification error bound b_p is negligible, then it is possible to neglect the last terms of Eq. (30), giving the simplified form as follows:

$$\|e(t)\|_\lambda \leq \gamma_1(t) \|e(t - T)\|_\lambda \quad (31)$$

The above expression guarantees the absolute convergence of error to zero as $t \rightarrow \infty$.

3. Simulation and experiments

In this section, the effectiveness of the proposed GFPC is demonstrated by simulations.

Example 1. In this example, we apply the proposed method to control the following second-order non-linear system with a saturation area of 0.7 and a dead zone of 0.07. The transfer function of the plant is

$$G(s) = \frac{20}{1.6s^2 + 4.4s + 1} \quad (32)$$

The system sampling period T is 0.01 s. Due to the large number of the system parameters in a GFPC, it is difficult to find a proper and dynamic step from all integer numbers by trial and error. So we limit the range to three prediction steps for the convenience of investigation. In general, the negative-step value can be selected from $[-4, -2]$, the small positive-step value can be selected from $[1, 10]$ and the big positive-step value can be selected from $[5, 20]$.

So there are 480 feasible solutions. In order to obtain an optimal solution, we define a cost function as follows:

$$f = \max \left[\exp \left(-\frac{ST}{3} \right) \exp(-OS) \right] \quad (33)$$

where ST is the settling time and OS is the max overshoot.

If the final solution f is obtained, it can provide the controlled system with a high overall performance. In this paper, the dynamic prediction mode performs the big positive-step $P_3 = 10$, small positive-step $P_2 = 2$ and negative-step $P_1 = -2$ mode based on the deviation between the set value $r(t)$ and the sensor output value $y(t)$ by simulation and calculation. So the switching mechanism is defined as follows:

$$p = \begin{cases} p_1 = -2 & \text{if } e(t) = r(t) - y(t) > 0.6 \\ p_2 = 2 & \text{if } 0.1 < e(t) = r(t) - y(t) < 0.6 \\ p_3 = 10 & \text{if } e(t) = r(t) - y(t) < 0.1 \end{cases}$$

By using Eq. (14), we obtain the grey prediction values and send them to the fuzzy controller in order to control the non-linear plant via a communication network. In the fuzzy controller part, we set $k_{e^*} = 60$, $k_{\Delta e^*} = 2.5$ and $k_u = 0.8$. The membership functions of input and output are shown in Fig. 2(a) and (b), respectively.

The control rules are shown in Table 1.

Fig. 3 shows the unit step response of the system obtained in simulation by using GFPC (the dotted line), TFC (the solid line), and traditional PID control with $K_p = 5$, $k_i = 0.1$, $k_d = 0.01$ (the dashed line), respectively.

The performance indices with these three different methods are shown in Table 2. As can be seen, the proposed method cannot only reduce the system overshoot efficiently but also maintain the characteristic of the shorter settling time of the system.

Example 2. In this example, we use the same controller to control the same plant as Example 1 via a communication network for showing the efficiency and robustness further. The structure of the proposed method is shown in Fig. 4.

It is well-known that the network-induced delay is brought into the control systems along with the inserted communication network, which not only prevents us from applying some conventional theorem to NCSs, but also brings many unstable

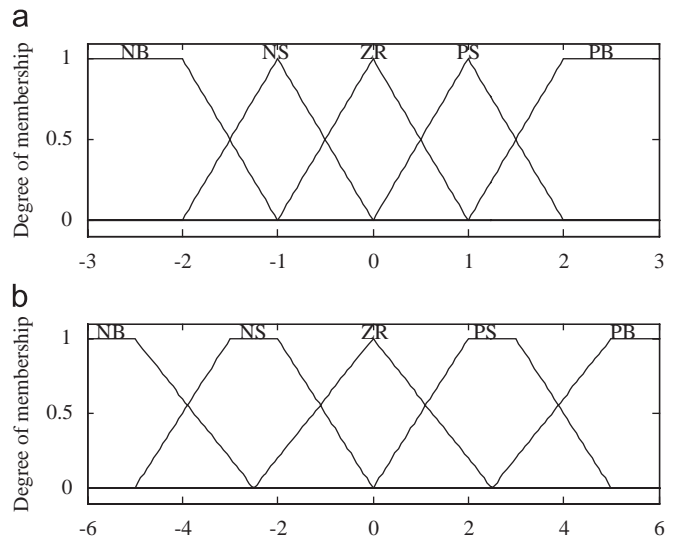


Fig. 2. Membership functions of fuzzy control system: (a) membership function of input fuzzy sets (e^* and Δe^*) and (b) membership function of output fuzzy sets (Δu).

Table 1
Fuzzy reasoning rule (U)

ΔE	E				
	NB	NS	ZR	PS	PB
NB	PB	PB	PS	PS	ZR
NS	PB	PS	PS	ZR	ZR
ZR	PS	PS	ZR	ZR	NS
PS	PS	ZR	ZR	NS	NS
PB	ZR	ZR	NS	NS	NB

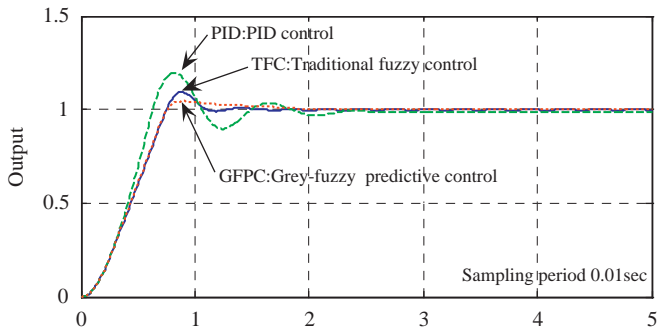


Fig. 3. Simulation results of the given system.

Table 2
The performance indices with three different methods

Performance	Settling time (s)	Overshoot (%)
PID	2.21	19.58
TFC	1.06	9.26
GFPC	0.92	3.90

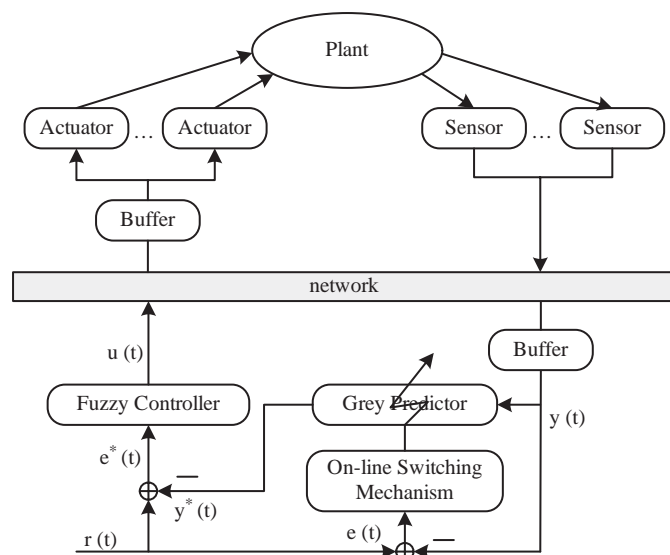


Fig. 4. GFPC in networked control systems.

factors that degrade the stability and control performance of the system [7,10,21].

In Fig. 4, we use buffers to change random time delays into constant time delays. The size of each buffer should be equal to the length of the signal data from the zero-step to the maximum-step network delay. In this way, the random network delay can be

treated as a constant delay after the buffer. This implies that if the transmission delay of the data on the network is less than the maximum delay, they have to stay in the buffer until the maximum delay is reached. For the convenience of investigation, we make the following assumptions [8,11,17]:

- (1) the upper bounds of the communication delays in the forward and backward channels are k and f of sampling period T , respectively;
- (2) the data transmitted through the communication network have a time stamp so that the packets arrive at the grey prediction node in a correct order;
- (3) there are no data packets lost;
- (4) the output of the plant is transmitted with one data packet. That is, a single packet is enough to transmit the plant output at every sampling period.

The simulation results are shown in Figs. 5 and 6, respectively. The dotted line is the simulation curve of GFPC, the solid line is the simulation curve of TFC, and the dashed line is the simulation curve of the traditional PID controller.

In Fig. 5, the upper bounds of the communication time-delays in the forward and backward channels are equal to the system sampling period (i.e. $k = f = T$), and in Fig. 6 the upper bounds of the communication time delays in the forward and backward channels are three times of system sampling period (i.e. $k = f = 3T$). The performance indices with these three different methods in NCSs are shown in Table 3.

As can be seen from the simulation results, two experiments with different time delays in forward and backward channels were conducted to test the GFPC design. By simulating a non-linear plant, we successfully improve the system control performances and robustness. The results are better than the ones that the traditional fuzzy and PID control strategies can provide.

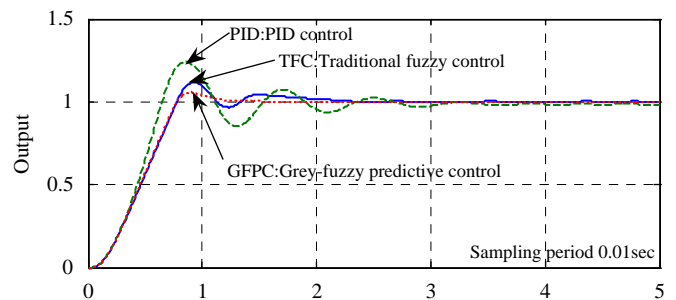


Fig. 5. Simulation results with the upper bounds of forward and backward channels $k = f = T$ in NCSs.

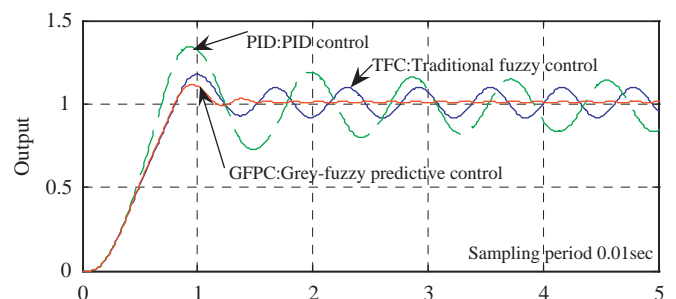


Fig. 6. Simulation results with the upper bounds of forward and backward channels $k = f = 3T$ in NCSs.

Table 3
The performance indices with three different methods in NCSs

Performance	Settling time (s)	Overshoot (%)
$k = f = T$		
PID	2.56	24.10
TFC	1.95	11.68
GFPC	1.1	5.76
$k = f = 3T$		
PID	∞	34.26
TFC	∞	17.73
GFPC	1.46	11.37

4. Conclusions and future work

This paper describes the design of a novel grey–fuzzy predictive controller, which combines grey and fuzzy theory with the on-line rule switching mechanism. Three forecasting modes are obtained, namely, big positive-step forecasting mode, small positive-step forecasting mode and negative-step mode. When the system error is large, the negative-step mode is used to increase the upward momentum of the output curve for shortening the settling time. When the system error is small, the positive-step mode is used to prevent the overshooting. The control signal is obtained according to the complementary behavior of the distinct modes. Simulation results indicate that the precision and robustness of the proposed GFPC method is better than other TFC methods for both NCSs and non-NCSs.

However, the proposed method is only applicable for a class of fuzzy logic controllers with symmetric and monotonic rule tables which are equivalent to a linear state feedback controller. How to choose the proper switching value, reduce conservatism and make the results satisfy other fuzzy controllers is one of the most important issues to be investigated in the future.

Acknowledgments

We would like to thank Prof. Guosen He for his support. This research is financially supported by National Natural Science Foundation of China under Grants 60774059 and 60834002, Key Project of Science and Technology Commission of Shanghai Municipality under Grants 061107031, 06ZR14131, and 061111008, Sunlight Plan Following Project of Shanghai Municipal Education Commission, and Shanghai Leading Academic Disciplines under Grant T0103.

References

- [1] Z.M. Chen, H.X. Lin, Q.M. Hong, The design and application of a genetic-based fuzzy gray prediction controller, *J. Grey Syst.* 1 (1) (1998) 33–45 (in Chinese).
- [2] B. Cheng, The grey control on industrial process, *J. Huangshi Coll.* 1 (1986) 11–23 (in Chinese).
- [3] F. Cupertino, V. Giordano, D. Naso, L. Delfino, Fuzzy control of a mobile robot, *IEEE Rob. Autom. Mag.* 13 (2006) 74–81.
- [4] J. Deng, Introduction to grey system theory, *J. Grey Syst.* (1) (1989) 1–24.
- [5] J. Deng, Grey Predicting and Grey Decision-making, Huazhong University of Science and Technology Press, Wuhan, China, 2002 (in Chinese).
- [6] C.C. Ding, K.T. Lee, C.M. Tsai, T.L. Huang, Optimal design for power system dynamic stabilizer by grey prediction PID control, in: Proceedings of IEEE International Conference on Industrial Technology, 2002, pp. 279–284.
- [7] M.R. Fei, J. Yi, H.S. Hu, Robust stability analysis of a class of uncertain nonlinear networked control system, *Int. J. Control Autom. Syst.* 4 (2) (2006) 172–177.
- [8] M.R. Fei, X.B. Zhou, T.C. Yang, Y.M. Tan, H.S. Wang, An initial study of gain-scheduling controller design for NCS using delay statistical model, *Lect. Notes Artif. Intell.* 4114 (2006) 1049–1060.
- [9] H.M. Feng, C.C. Wong, An on-line rule tuning grey prediction fuzzy control system design, in: Proceedings of the 2002 International Joint Conference on Neural Networks, 2002, pp. 1316–1321.
- [10] S.S. Hu, Q.X. Zhu, Stochastic optimal control and analysis of stability of networked control systems with long delay, *Automatica* 39 (11) (2003) 1877–1884.
- [11] G.P. Liu, S.C. Chai, D. Rees, Networked predictive control of internet/intranet based systems, in: Proceedings of the 25th Chinese Control Conference, 7–11 August, 2006, Harbin, Heilongjiang, pp. 2024–2029.
- [12] R.C. Luo, Tse Min Chen, Autonomous mobile target tracking system based on grey–fuzzy control algorithm, *IEEE Trans. Ind. Electron.* 47 (4) (2000) 920–931.
- [13] H. Marzi, Fuzzy control of an inverted pendulum using AC induction motor actuator, in: IEEE International Conference on Computational Intelligence for Measurement Systems and Applications, La Coruna, Spain, July 2006, pp. 109–114.
- [14] W. Qiau, M. Muzimoto, PID type fuzzy controller and parameter adaptive method, *Fuzzy Sets Syst.* 78 (1996) 23–25.
- [15] M. Sugeno (Ed.), *Industrial Applications of Fuzzy Control*, Elsevier Science Publishers, Amsterdam, 1985.
- [16] L.X. Wang (Ed.), *Adaptive Fuzzy Control Systems: Design and Stability Analysis*, National Defence Industry Press, Beijing, China, 2000 (in Chinese).
- [17] L.S. Wei, M.R. Fei, A real-time optimization grey prediction method for delay estimation in NCS, in: The Sixth IEEE International Conference on Control and Automation May 30–June 1, 2007, Guangzhou, China, pp. 514–517.
- [18] C.C. Wong, W.C. Liang, H.M. Feng, D.A. Chiang, Grey prediction controller design, *J. Grey Syst.* 2 (1998) 123–131.
- [19] H. Ying, Analytical structure of a fuzzy controller with linear control rules, *Inf. Sci.* 81 (1994) 213–227.
- [20] G.R. Yu, C.W. Chuang, R.C. Hwang, Fuzzy control of brushless DC motors by gray prediction, in: Ninth IFSA World Congress and 20th NAFIPS International Conference, vol. 5, 2001, pp. 2819–2824.
- [21] L. Zhang, D. Hristu-Varsakelis, Communication and control co-design for networked control systems, *J. Automatica* 42 (6) (2006) 953–958.



Lisheng Wei received M.S. degrees in Navigation, Guidance and Control from the 061 Base of China Aerospace Science and Industry Corporation in 2004. Now he is a Ph.D. candidate in control theory and control engineering of Shanghai University. His current research interests are in the areas of network control, intelligent control, multi-variable system identification.



Minrui Fei received his B.S. and M.S. degrees in Industrial Automation from the Shanghai University of Technology in 1984 and 1992, respectively, and his Ph.D. degree in Control Theory and Control Engineering from Shanghai University in 1997. Since 1998, he has been a Professor and Doctoral Supervisor at Shanghai University. His current research interests are in the areas of intelligent control, complex system modeling, networked control systems, field control systems, etc.



Huosheng Hu received his Ph.D. degree from Oxford University, UK. He is a Professor in Computer Science at the University of Essex, UK, leading the Human Centred Robotics Group. He is also a Zhiqiang Professor in Shanghai University. His research interests include biologically inspired robotics, service robots, human–robot interaction, evolutionary robotics, data fusion, artificial life, embedded systems, pervasive computing and RoboCup. He is also a Chartered Engineer, a senior member of IEEE and a member of IEE, AAAI, IAS, IASTED and ACM.